Mine Surveying

Chain Surveying

Chain Surveying

CHAINS

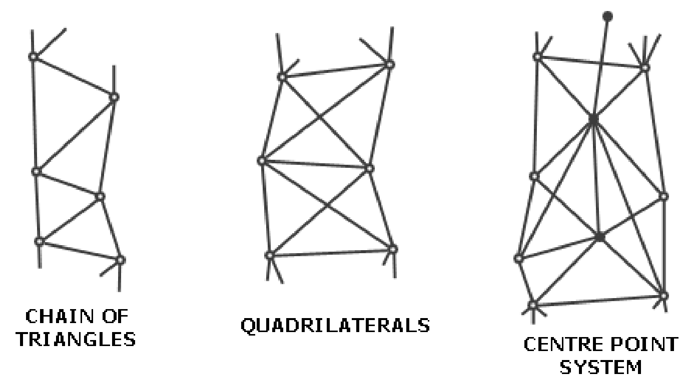
Gunter's Chain : The Gunter's chain is 66 ft. Long and is divided into 100 links each 0.66 ft. Long. It is very convenient for measuring distances in miles and furlongs and for measuring land when the unit of area is an acre, on account of its simple relation to the mile and the acre.

Revenue Chain : The revenue chain is commonly used for measuring fields in cadastral survey. It is 33 ft. Long divided into 16 links.

Engineers' Chain : The engineers' chain is 100 ft. Long and is divided into 100 links each one foot in length.

PRINCIPLE OF CHAIN SURVEYING

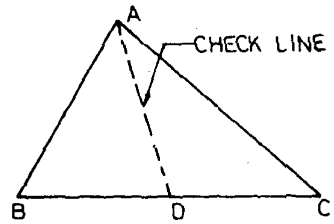
The principle of a chain survey is triangulation. It consists of the arrangement of framework of triangles, since a triangle is the only simple plane figure, which can be plotted from the lengths of its sides alone. The three sides of a triangle being equally liable to error, each of the three angles of a triangle should be nearly 60^o , i.e. the triangle should be equilateral.



The framework should therefore consists of triangles which are nearly equilateral as possible, such triangles being known as **well conditioned**. A triangle is said to be **well conditioned** or well proportioned when it contains no angle smaller than 30^o and no angle greater than 120^o.

Base Line : The longest of the chain lines used in making a survey is generally regarded as the base line. It is generally the most important line. It fixes up the directions of all other lines, as on the base line is built up the framework of a survey.

Check Line : A check line such as Ed (see figure below), also termed as a proof line, is a line joining the apex of a triangle to some fixed point on the side opposite, or a line joining some fixed points on any two sides of a triangle. A check line is measured to check the accuracy of the framework, as the length of a check line as measured on the ground should agree with its length on the plan.



SURVEY STATION AND MAIN STATION

A Survey Station is a point of Importance at the beginning and end of a chain line. There are two main types of stations namely **Main station and Subsidiary or Tie station**.

Main station

Main stations are the ends of the lines which command the Boundary of the survey, and the lines joining the main stations are called main Survey or Chain lines.

Subsidiary or tie station

Any Point selected on the main survey line where it is necessary to run the auxiliary lines to locate the interior details such as fences, hedges, buildings, etc., when they are at some distance from the main survey lines are known as Subsidiary or Tie stations .

Tie line

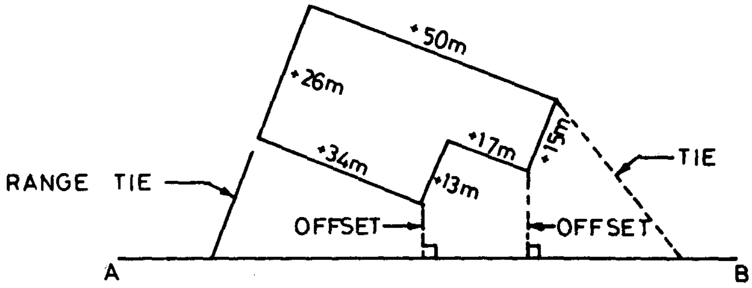
The lines joining such stations are known as tie line or subsidiary line. A tie line usually fulfils a dual purpose, viz. It checks the accuracy of the framework and enables the surveyor to locate the interior details which are far away from the main chain lines.

OFFSETS

In a survey the positions of the details such as boundaries, building, fences, rods, nallas etc are located with respect to the survey (r chain) lines by means of lateral measurements (i.e. distances measured from the chain lines) to such objects right or left of the chain lines. These lateral measurements are called offsets. There are two kinds of offsets :

1. perpendicular offsets
2. oblique offsets.

In the strict sense, offsets are always taken at right angles to the survey line. They are also called perpendicular or right-angled offsets.



The measurements which are not made at right angles to the survey line are called oblique offsets or *tie*-line offsets.

Taking Offsets

Every offset involves two measurements

1. the distance along the chain line called chainage, and
2. the length of the offset (perpendicular or oblique). These are taken and noted in a field book. This operation is known as taking offset.

RANDOM LINE

In a linear measurements when end stations are not visible from any intermediate point, random line method is used. In this method, a random line is drawn in the estimated direction upto the point from which the other end point is visible.

ERRORS AND MISTAKES IN CHAINING

Errors in chaining may be caused due to variation in temperature and pull, defects in instruments, etc. They may be either

- compensating, or
- cumulative

Compensating Errors

Errors which may occur in both directions (i.e. both positive and negative) and which finally tend to compensate are known as compensating errors.

These errors do not affect survey work seriously. They are proportional to , where L is the length of the line. Such errors may be caused by

- Incorrect holding of the chain,
- Horizontality and verticality of steps not being properly maintained during the stepping operation,
- Fractional parts of the chain or tape not being uniform throughout its length, and
- Inaccurate measurement of right angles with chain and tape.

Cumulative Errors

Errors which may occur in the same direction and which finally tend to accumulate are said to be cumulative. They seriously affect the accuracy of the work, and are proportional to the length of the line (L). The errors may be positive or negative.

Positive Errors When the measured length is more than the actual length (i.e. when the chain is too short), the error is said to be positive. Such errors occur due to

- The length of chain or tape being shorter than the standard length,
- Slope correction not being applied,
- Correction for sag not being made,
- Measurement being taken with faulty alignment, and
- Measurement being taken in high winds with the tape in suspension.

Negative Errors When the measured length of the line is less than the actual length (i.e. when the chain is too long), the error is said to be negative. These errors occur when the length of the chain or tape is greater than the standard length due to the following reasons.

- The opening of ring joints,
- The applied pull being much greater than the standard pull,
- The temperature during measurement being much higher than the standard temperature,
- Wearing of connecting rings, and
- Elongation of the links due to heavy pull.

TAPE CORRECTIONS

Following corrections are applied to the measured distances

Correction for Standardisation

The correction is required if the true length (actual length, on site length) of the tape is not equal to the nominal length (factory length, designated length).

Correction per tape length (C) = l' – l

where l is nominal(designated, factory length) length of the tape and l' is actual length(on site) of tape.

The correction is positive when the actual length (l') is greater than the nominal length (l) and vice versa.

Total correction in measured distance

$$C_a = \frac{(l' - l)}{l} \times L = \frac{C \times L}{l}$$

Correction for Slope

Correction $C_g = D - L = -L(1 - \cos\theta) = -2L \sin^2\theta/2$

where D = horizontal equivalent, L = slope distance and θ is angle of slope.

Alternatively $C_g = \sqrt{(L^2 - h^2)} - L$

where h is difference in elevation of the end points.

Correction for Pull

The correction for pull (C_p) is given by

$$C_p = \frac{(P - P_0)L}{AE}$$

where P is pull applied during measurement (N), P_0 is standard pull (N), L is measured length, A is cross sectional area of the tape and E is Young's modulus of elasticity (for steel $E = 2 \times 10^5$ N/mm^2 or 2×10^5 MPa).

If the pull applied at the tape is greater than the standard pull, the actual length (on site) of the tape is greater than the nominal length (factory length), and a *positive* correction is required.

Correction for Temperature

The temperature correction is given by

$$C_t = \alpha(T - T_0)L$$

where α is coefficient of linear expansion, T is mean temperature of the tape (^{0}C) and T_0 is standard temperature (^{0}C). The sign of C_t is directly given by above equation.

For steel tapes, $\alpha = 1.15 \times 10^{-15}$.

Correction for Sag

Correction for sag is given by

$$C_s = -\frac{l_1(wl_1)^2}{24P^2}$$

where w is weight of tape per unit length (N/m), P is applied pull (N) and l_1 is length of the tape suspended between the supports (m).

The above equation can also be written as

$$C_s = \frac{l_1 W_1^2}{24P^2}$$

where W_1 = total weight of the tape between supports.

The sag correction is *always* negative.

If the tape is suspended between the supports is pulled by a large force, there is a decrease in the sag and an increase in the length of the tape because of tension applied. Normal tension

(P_n) is the theoretical pull at which the pull correction is numerically equal to the sag correction.

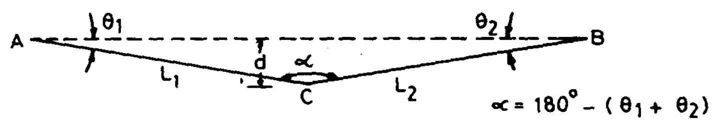
$$\frac{(P_n - P_0)l_1}{AE} = \frac{l_1 W_1^2}{24P_n^2}$$

or $$P_n = \frac{0.204 W_1 \sqrt{AE}}{\sqrt{p_n - P_0}}$$

This equation can be solved by trial and error.

Correction for Misalignment

If the survey line is not accurately ranged out, the error due to misalignment occurs. The measured distance is always greater than the correct distance, and hence the error is positive and the *correction is negative.*



The correction due to misalignment is given by

$$C_m = (L_1 \cos\theta_1 + L_2 \cos\theta_2) - (L_1 - L_2)$$

or $$C_m = \sqrt{L_1^2 + L_2^2 - 2L_1L_2\cos\alpha} - (L_1 + L_2)$$

or $$C_m = \left[\sqrt{L_1^2 - d^2} + \sqrt{L_2^2 - d^2}\right] - (L_1 + L_2)$$

Correction for Local Scale Factor

A short traverse on a plane surface can be easily represented on a plan with grid lines without any apparent distortion, but in the case of larger areas the curvature of the spherical surface of earth greatly affect the actual shape of the map. Objects on the curved surface of earth can not be represented on a flat sheet of paper in the same relative positions which they occupied on the globe. A slight difference between the grid distances occurs because the meridians converge and the grid lines are drawn parallel to one central meridian. Therefore, in precise surveys of larger areas the distances computed from coordinate will require a correction by a factor known as "local scale factor". The value of the local scale factor will vary with geographical position and depending upon the departure east or west from the central meridian.

Example

What is normal pull ? Show that for a chain of 3 mm^2 area and 0.48 Kg weight (E = 2x10^6 Kg/cm^2) Standardised at 8 Kg tension, the normal pull is 12 Kg.

Solution

The pull which when applied to a tape supported in air over two ends equalizes the correction due to pull and the correction due to sag is known as normal pull.

The correction for pull = $c_1 = (P - P_0)/AE$ (+ ve)

The correction for sag $c_2 = LW^2/24P^2$ (-ve)

equating numerically the two equations , we get

$$(P-P_0)/AE = LW^2/24P^2$$

$$P = (0.204W\sqrt{AE} / \sqrt{(P-P_0)}$$

The value of normal pull `P' is determined by trial and error with the help of above equation.

Area $A = 3\ mm^2 = 0.03\ cm^2$

$E = 2 \times 10^{6}\ Kg/cm^2$

W - 0.48 Kg, P0 = 8 Kg

to prove P = 12 Kg

$$P = 0.204x0.48\sqrt{(0.03x2x10^6)} / \sqrt{(12 - 8)}$$

$= 11.993 = 12$ Kg Hence Proved.

Example

A steel tape 30 m long was standardised with a pull of 65 N. If the pull at the time of measurement was 45 N, find the correction per tape length. The tape weighs 10 N. Take E = $2 \times 10^5\ N/mm^2$ and weight of 1 m^3 of steel as 77.10 kN.

Solution

Let A be the cross sectional area of the tape in mm^2.

Weight of 1 mm^2 steel = $77.10 \times 10^3/10^9 = 77.1 \times 10^{-6}$ N

Therefore $A \times (30 \times 10^3) \times 77.1 \times 10^{-6} = 10$

or $A = 4.323\ mm^2$

Now $$C_p = \frac{(45-65)}{4.323 \times 2 \times 10^5} = -6.93 \times 10^{-4}\ m = -0.693\ mm$$

Example

Determine the sag correction for a 30 m steel tape under a pull of 80 N in 3 bays of 10 m each. The area of the cross section of the tape is 8 mm^2 and the unit weight of steel may be taken as 77 kN/m^3.

Solution

Weight of the tape per metre = $(77 \times 10^3/10^9) \times 8 \times 10^3$ = 0.616 N/m

Now sag correction per bay will be

$$\frac{l_1(wl_1)^2}{24P^2} = -\frac{10x(0.616x10)^2}{24\,x\,(80)^2} = -2.470\,x\,10^{-3}\text{ m}$$

Total correction for 3 bays = $-3 \times 2.470 \times 10^{-3} = -7.411 \times 10^{-3}$ m

Example

A metallic tape originally 20 m is now found to be 20.2 m long. A house, 30 m x 20 m is to be laid out. What measurements must be made using this tape ? What should the diagonal read?

Solution

True distance = length to be measured x (Incorrect length of tape L'/Correct length of tape L)

30 = Length to be measured x 20.2 / 20

Length to be measured = 30 x 20/20.2 = 29.802 m

Hence measurements to be made with 20.2 m tape instead of 20 m tape are 29.703 x 19.802m **Ans**.

Diagonal measurement = $\sqrt{(29.703)^2 + (19.802)^2}$ = 35.7 m **Ans**.

Problem

A 20 m chain which was 20 cm too short was used to measure a line and the result was 196.1 m. What was the true length of the line ?

Ans.: 194.137

Example

An mining land was measured with an incorrect 30 m chain and a plan was drawn. From these measurements the area on the plan was measured and calculated and was found to be 16.25 km^2. Find to correct area of the mining land, if the length of the chain was 30.06 m.

Solution

True area = $(L/30)^2 \times 16.25\text{ km}^2 = (30.06/30)^2 \times 16.25\text{ km}^2 = 16.315\text{ km}^2$ **Ans**.

Problem

An area was surveyed by chain and tape and plotted in a scale 1:1000. When it is necessary to measure the area it is found the map has shrunk by 1 % both ways. The planimeter reading of the area is 5321 sq. m What is the correct area of the plot.

Ans : 53199358 m^2.

Example

A steel tape of nominal length 30 m was used to measure a line AB by suspending it between supports. If the measured length was 29.861 m when the slope angle was $3^0 45'$, and the mean temperature and tension applied were respectively 10^0C and 100 N, determine the corrected horizontal length.

The standard length of the tape was 30.004 m at 20^0C and 44.5 N tension. The tape weighted 0.16 N/m and had a cross-sectional area of 2 mm^2. E = 2 x 10^5 N/mm^2. α = 1.12 x 10^{-5} per ^{0}C.

Solution

Slope correction

$$= - L(1 - \cos\theta)$$

$$= - 29.861 (1 - \cos 3^0 45')$$

$$= - 0.064 \text{ m}$$

Standardisation correction

$$= Lx\left(\frac{l' - l}{l}\right)$$

$$= 29.861\left(\frac{30.004 - 30.000}{30.000}\right) = +0.004\,\text{m}$$

Temperature correction

$$= \alpha(T - T_0)L$$

$$= 1.12x10^{-5}(10 - 20)x29.861 = -0.003\,\text{m}$$

Pull correction

$$= \frac{(P - P_0)L}{AE}$$

$$= \frac{(100 - 44.5)x29.861}{2x2x10^5} = +0.004\,\text{m}$$

Sag correction

$$= \frac{w^2 l_1^3}{24P^2}\cos^2\theta$$

$$= \frac{-(0.16)^2 x (29.861)^3 x \cos^2 3^0 45'}{24x(100)^2} = -0.003\,\text{m}$$

Total correction = - 0.064 + 0.004 - 0.003 + 0.004 - 0.003 = - 0.062

Correct horizontal distance = 29.861 – 0.062 = 29.799 m

Problem

A steel tape 30 m long standardized at 10^oC with a pull of 10 kg was used for measuring a base line. Find the correction per tape length, if at the time of measurement the temperature was 22^oC and the pull exerted 15 kg. Weight of steel/cm^3 = 7.75 g. Weight of tape 0.68 kg, E = 2.11×10^6 kg/cm^2, α = 12×10^{-6} m /^oC.

Ans.: 0.0041 m (+ve)

Example

Find the maximum length of offset so that the displacement of a point on the paper should not exceed 0.025 cm, given that the offset was laid out 3^o from its true direction and the scale was 10 m to 1 cm.

Solution

Let l = the limiting length of offset in m.

α = the angular error in direction.

Then,

Displacement of the point on the ground = l sin α = l sin 30^o

Since the scale is 10 m to 1 cm, its displacement on the paper

= l sin 3^o/10 cm and this should equal 0.025 cm.

l sin 3^o/10 = 0.025 or l = 4.78 m.

FIELD WORK

Equipments

- a chain and to arrows
- a 20 m metallic tape
- Ranging rods 12 nos.
- An offset rod
- An optical square or a cross staff
- A plumb bob
- A survey field book
- Pegs
- 2 pencils of good quality and penknife
- A good field glass
- Sundries such as chalk, hammer, nails etc.

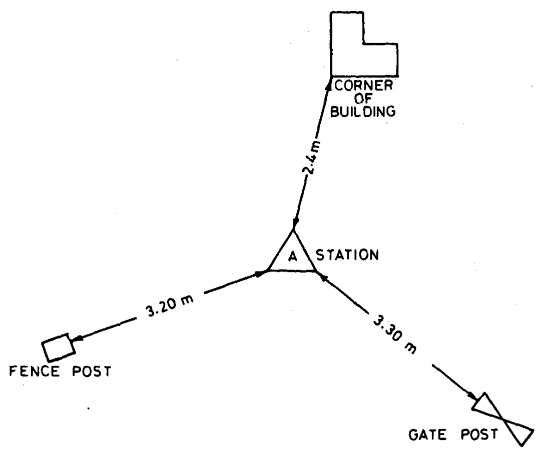
Executing Chain Surveying in the Field

A chain survey is executed in the following steps:

1. **Reconnaissance**. The preliminary inspection of the area to be surveyed is called reconnaissance. It is essential that the surveyor should have thorough knowledge of the ground to be surveyed and its principal features. On arriving at the ground, the surveyor should, therefore, walk over the whole area and thoroughly examine the ground so as to decide upon the best possible arrangement of the work. By careful reconnaissance the surveyor obtains a fairly intimate knowledge of the shape and extent of the area to be surveyed, which will help him to form a fairly accurate idea of the difficulties in the work, time required for the work and the most suitable positions for the main lines free from obstacles on the level ground.

2. **Marking stations**. Having completed the reconnaissance, survey stations should be marked on the ground as detailed below so that they can be readily discovered when required : (i) By fixing vertically a ranging rod at each stations if the survey is of only a temporary character and can be finished in a single day. (ii) By driving in firmly a wooden peg at each station with about 2.5 cm to 4 cm standing out of the ground, if the survey is extensive.

3. **Reference Sketches**. After the stations are marked, they should be referenced, i.e. located by measurements, called ties, taken from three permanent points which are definite and easily recognised see following figure.



To draw a location sketch, the surveyor, facing the north direction, should draw the direction of the north line and show the relative positions of the reference points and the station, and record the tie measurements between the arrows, the arrow heads touching the station point and the reference points as in figure 4.

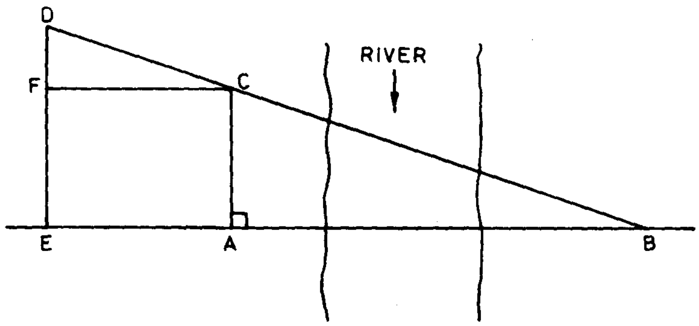
4. **Chaining**. Having finished the preliminary work, chaining may be commenced from the base line and carried throughout all the lines of the framework as continuously as possible. Offsets are taken to the adjacent objects and we booked in field book.

OBSTACLES IN CHAINING

When obstacle is a river

1. Select two points A and B on the chain line on either side of the obstacle.

2. Select another point E on the chain line.
3. Erect a perpendicular CA at A on AB.



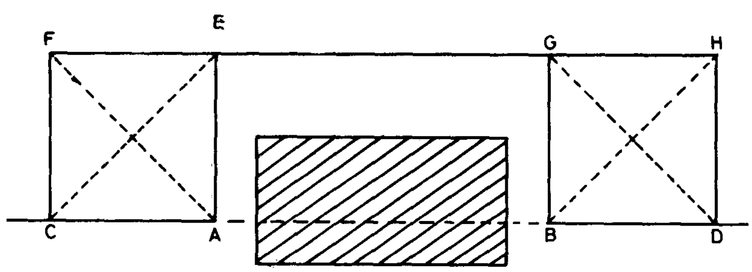
4. Erect a perpendicular ED at E on AB such that D, C and B are in one line.
5. From C draw a line CF parallel to AE, meeting DE at F.
6. Measure AE, CA, ED and AC.
7. Calculate AB as

$$AB = \frac{AE \times CA}{ED - AC}$$

When obstacle is a building

When we come across a building which obstruct both chaining abd ranging. This obstacle may be overcome by the following method.

1. Select two points A and C on the chain line before the obstacle (see figure below).
2. Erect perpendicular AE and CF of equal lengths. Check that the lengths of diagonal AF and CE are equal to ensure corrections.



3. Join F and E and extend the line FE to point G beyond the obstacle. Erect a perpendicular GB on FG such that GB = AE.
4. Extend the line FG to H, and erect a perpendicular HD on FH and make HD = AE.
5. Join points B and D. Line BD is the extension of the chain line CA beyond the obstacle.

Obviously AB = EG

Example (AMIE W 94)

A survey line PQ intersects a tall building. To continue the line PQ, QR of length 120 m was set out at right angles to PQ. From R two lines, RS and RT, making angles 45^o and 60^o with RQ, were ranged. Find the lengths of RS and RT in order that the stations `S' and `T' may be in PQ produced, and length of `QS' past the building.

Solution

From Δ QRS

$\tan 45^o = QS / QR$

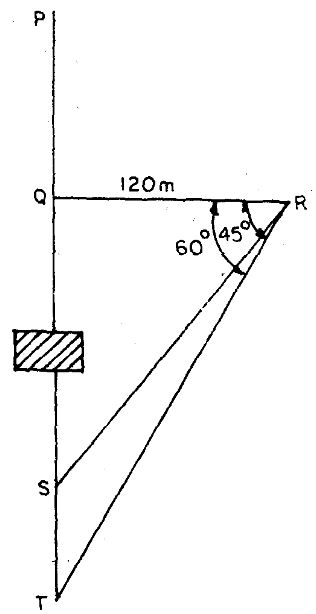
$QS = \tan 45^o \times QR = 120$ m

$RS = \sqrt{(QR)^2 + (QS)^2} = \sqrt{(120)^2 + (120)^2} = 169.71$ m

From Δ QRT, $\tan 60^o = QT / QR$

$QT = QR \tan 60^o = 120\sqrt{3}$ m

$RT = (\sqrt{(120\sqrt{3})^{2)}}) + (120)^2 = 240$ m

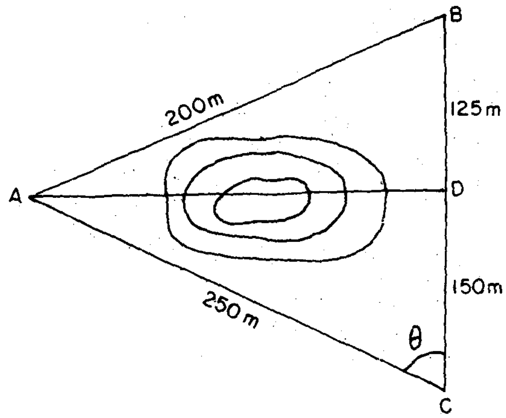


∴ Length of QS = 120 m, RS = 169.71 m, RT = 240 m

Example

In passing an obstacle in the form of a pond, stations A and D, on the main line, were taken on the opposite sides of a pond. On the left of AD a line AB, 200 m long, was laid down and a second line AC, 250 m long was, ranged on the right of AD, the point B, D and C being in the same straight line. BD and DC were then chained and found to be 125 m and 150 m respectively. Find the length of AD.

Solution



From Δ ACB,

$$\cos\theta = (CA^2 + CB^2 - BA^2)/(2.AC.CB)$$

$$= (250^2 + 275^2 - 200^2)/(2x250x275) \quad (1)$$

Also in Δ ACD,

$$\cos\theta = (AC^2 + CD^2 - AD^2)/(2.AC.CD)$$

$$= (250^2 + 150^2 - AD^2)/(2x250x150) \quad (2)$$

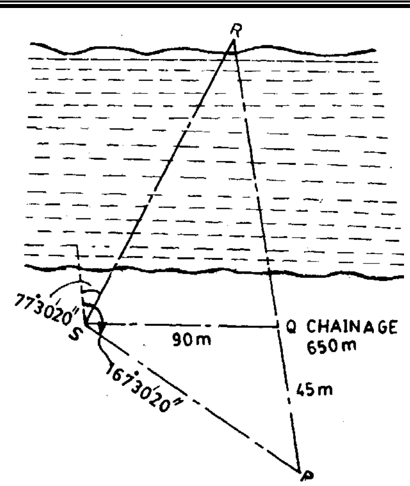
Comparing (1) and (2),

AD = 177.42 m **Ans**.

Example

A chain line PQR crosses a river, Q and R being on the near and distant banks respectively. A perpendicular QS, 90 m long, is set out at Q on the left of the chain line. The respective bearing of R and P taken at S are $77^o30'20''$ and $167^0 30'20''$. Find the chainage of R given the PQ is 45 m and the chainage of Q is 650 m.

Solution



Chainage of Q = 650 m

Length of PQ = 45 m

Bearing of SP = $167^o30'20''$

Bearing of SP = $77^o30'20''$

$\angle$ RSP = Bearing of SP - Bearing of SR = $167^o30'20''$ - $77^o30'20''$ = 90^o

From similar triangles RSQ and SQP, we get

RQ/SQ = SQ/QP

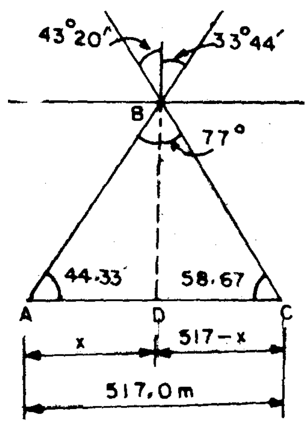
RQ = SQ x SQ/QP = 90 x 90/45 = 180 m

Chainage Of R = Chainage of Q + Length QR = 650 + 180 = 830 m **Ans.**

Example

P and Q are two points 517 m apart on the same bank of a river. The bearing of a tree on the other bank observed from P and Q are N $33^0$40' E and N43^0 20' W respectively. Find the width of the river if the bearing of PQ is N78E.

Solution



From Δ ABD,

BD = s tan 44.33 = 0.976 x (1)

From Δ BDC,

BD = (517 - x) tan 58.67 (2)

Now, 0.976 x = 849.31 - 1.64 x

$\therefore$ x = 324.66

Hence, Width of river = 0.976 x 324.66 = 316.869 m **Ans.**

Example

In chaining an area containing a pond, two points C and D were selected on either sides of chain station A such that A, C and D points lie on a line. The points B which is on the other side of pond is on the chain line AB. If distances AC, AD, BC and BD are 35 m, 45 m, 100 m, and 95 m respectively, determine the length of the chain line AB and the angles which the inclined line CD makes with the chain line AB.

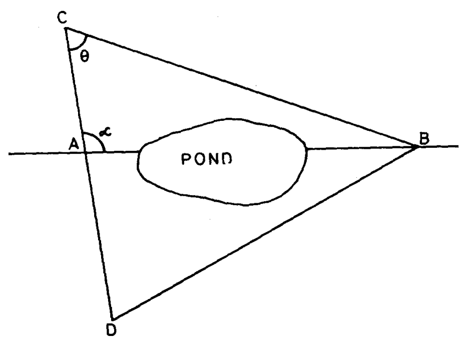
Solution

See following figure.

Let the angle BCD be equal to θ.

In the triangle ABC

$$\cos\theta = \frac{(100)^2 + (35)^2 - (AB)^2}{2 \times 100 \times 35} \quad (1)$$



In the triangle BCD

$$\cos\theta = \frac{(100)^2 + (80)^2 - (95)^2}{2 \times 100 \times 80}$$

or $\theta = 62.55^0$

From (1) $\cos\theta = \frac{11225 - (AB)^2}{7000}$

or $0.4609 = \frac{11225 - (AB)^2}{7000}$

or AB = 89.44 m

Let α be the angle which the line CD makes with the line AB. From the triangle ABC by cosine law.

$$\cos\alpha = \frac{(35)^2 + (89.44)^2 - (100)^2}{2 \times 35 \times 89.44}$$

∴ $\alpha = 97.11^0$

Problem

There is an obstacle in the form of a pond, on the main chain line. Point C and D were taken on the opposite sides of the pond. On left of CD, a line CE was laid out of 1000 m length and a second line CF = 800 m was laid on the right of CD such that E, D and F are in the same straight line ED and DF were measured and found to be 600 m and 560 m respectively. Find out the length CD which is obstructed.

Ans.: CD = 619.2 m

Problem

A chain line AB intersects a building. From a point C near one end of the building, a perpendicular CE = 400 m is set out from E, two lines ED and EF are laid down at angles of 45^0 and 60^0 with CE respectively. Determine the lengths ED and EF and the obstructed distance CD. Points D and F lies on the chain line.

Ans.: ED = 565.6 m, EF = 800 m

CHAINING ON A SLOPING GROUND

Chaining is a process if measuring distance between two stations in the ground. Chaining on sloping ground can be divided into two methods :

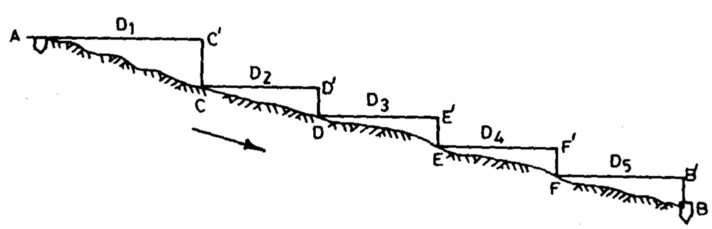
(i) Direct Method(stepped method)

(ii) Indirect method (By measuring along slope)

a) By measuring along slope and measuring vertical angles
b) Hypotenuse allowance method
c) Pythagoras theorem method

Direct Method (By Stepping)

In this method, the horizontal distance are measured in short horizontal lengths by stepping method.



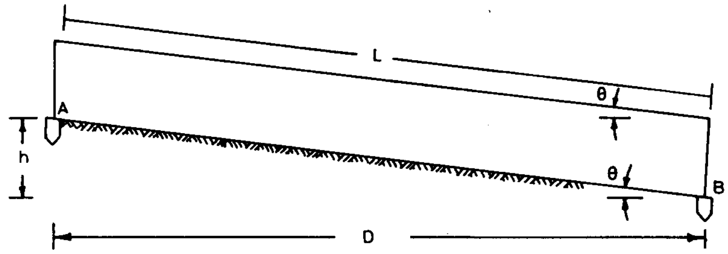
The tape is held horizontal (AC') and the point C is found by suspending a plumb job.

D = AB = AC' + CD' + DE' + EF' + FB'

V = C'C + D'D + E'E + F'F + B'B

Indirect Method (By measuring along slope)

In this method, the horizontal distance is obtained by measuring along slopes and obtaining vertical angles by a clinometer.



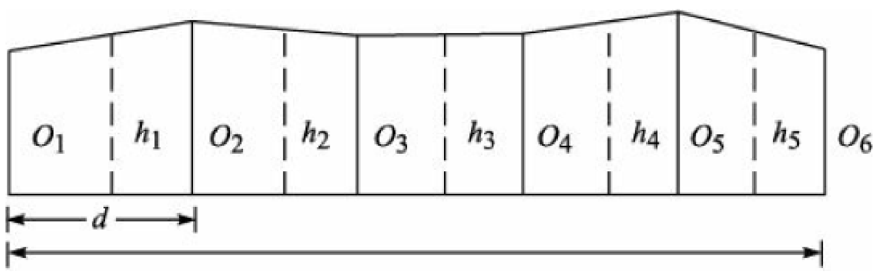
$D = L_1 Cos\theta_1 + L_2 Cos\theta_2$

$V = L_1 Sin\theta_1 + L_2 Sin\theta_2$

where L_1, L_2 etc are the inclined distances, and θ_1, θ_2 etc. are the angles of slopes.

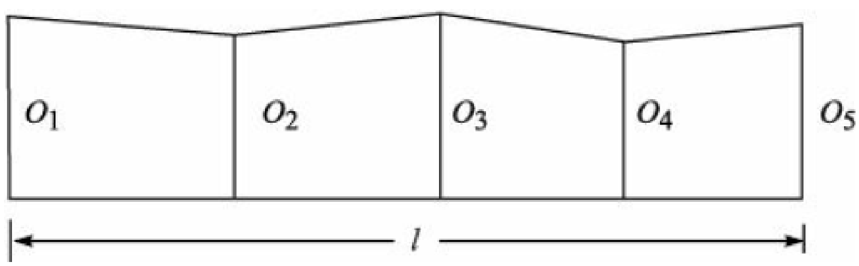
COMPUTATION OF AREA

Mid-ordinate Rule



Area = Common distance × sum of mid-ordinates

Average-ordinate Rule



Area = (sum of ordinates/no. of ordinates) x length of base line

Example

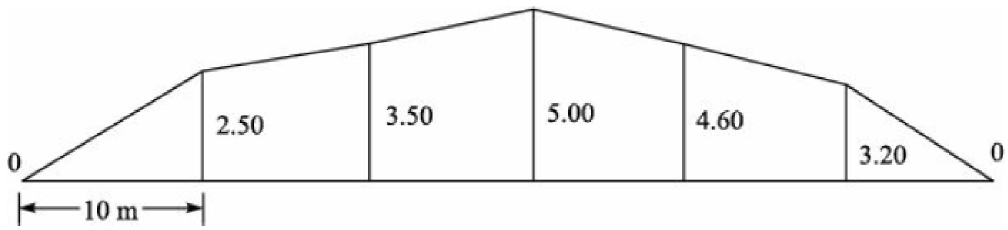
The following offsets were taken from a chain line to an irregular boundary line at an interval of 10 m:

0, 2.50, 3.50, 5.00, 4.60, 3.20, 0 m

compute the area between the chain line, the irregular boundary line and the end offsets by (i) the mid-ordinate rule (ii) average ordinate rule.

Solution

Figure



Mid ordinate rule

$h_1 = (0 + 2.50)/2 = 1.25$ m; $h_2 = (2.50 + 3.50)/2 = 3.00$ m;

$h_3 = (3.50 + 5.00)/2 = 4.25$ m; $h_4 = (5.00 + 4.60)/2 = 4.80$ m

$h_5 = (4.60 + 3.20)/2 = 3.90$ m; $h_6 = (3.20 + 0)/2 = 1.60$ m

Required area = 10(1.25 + 3.00 + 4.25 + 4.80 + 3.90 + 1.60) = 10 × 18.80 = 188 m^2

Average ordinate rule

Here, d = 10 m and n = 6 (no. of divisions)

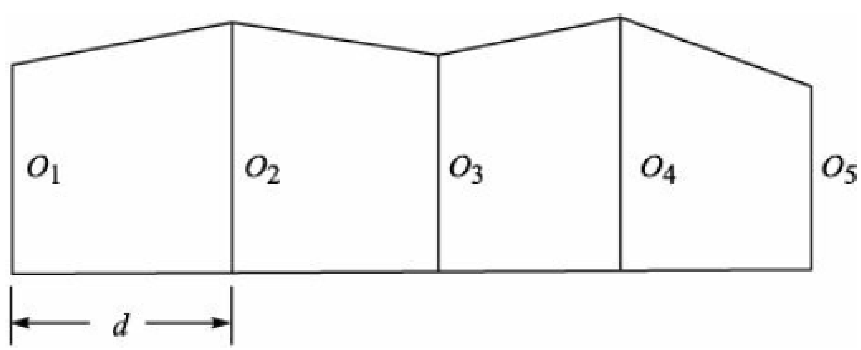
Base length = 10 × 6 = 60 m

Number of ordinates = 7

Required area = $60\,x\left(\frac{0+2.50+3.50+5.00+4.60+3.20+0}{7}\right)$ = 161.14 m^2

Trapezoidal Rule

While applying the trapezoidal rule, boundaries between the ends of ordinates are assumed to be straight. Thus the areas enclosed between the base line and the irregular boundary line are considered as trapezoids.



Total area = (d/2)(1st ordinate + last ordinate + 2 x sum of other ordinates)

Simpson's rule

Total area = (d/3)(1st ordinate + last ordinate + 4 x sum of even ordinates + 2 x sum of remaining odd ordinates)

Limitation This rule is applicable only when the number divisions is even, i.e. the number of ordinates is odd.

☞ Sometimes one, or both, of the end ordinates may be zero. However, they must be taken into account while applying these rules.

Example

The following offsets were taken at 15 m intervals from a survey line to an irregular boundary line:

3.50, 4.30, 6.75, 5.25, 7.50, 8.80, 7.90, 6.40, 4.40, 3.25 m

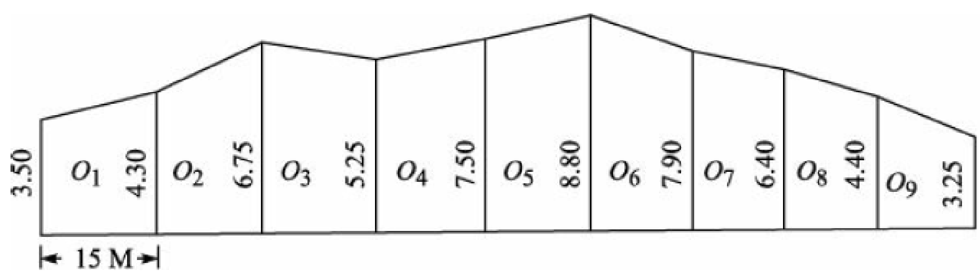
Calculate the area enclosed between the survey line, the irregular boundary line, and the first and last offsets, by

(a) The trapezoidal rule

(b) Simpson's rule

Solution

Figure



Trapezoidal Rule

Required area

$$=(15/2)\{3.50 + 3.25 + 2(4.30 + 6.75 + 5.25 + 7.50 + 8.80 + 7.90 + 6.40 + 4.40)\} = (15/2)\{6.75 + 102.60\} = 820.125 \text{ m}^2$$

Simpson's Rule

If this rule is to be applied, the number of ordinates must be odd. But here the number of ordinate is even (ten).

So, Simpson's rule is applied from O_1 to O_9 and the area between O_9 and O_{10} is found out by the trapezoidal rule.

$$A_1 = (15/3)\{3.50 + 4.40 + 4(4.30 + 5.25 + 8.80 + 6.40) + 2(6.75 + 7.50 + 7.90)\}$$

$$= (15/3)\{7.90 + 99.00 + 44.30\} = 756.00 \text{ m}^2$$

$$A_2 = (15/2)\{4.40 + 3.25\} = 57.38 \text{ m}^2$$

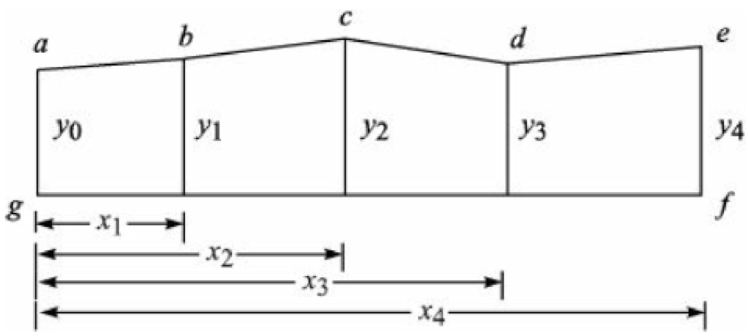
Total area = $A_1 + A_2$ = 756.00 + 57.38 = 813.38 m^2

Coordinate method

When offsets are taken at very irregular intervals, then the application of the trapezoidal rule and Simpson's rule is very difficult. In such a case, the coordinate method is the best.

Procedure

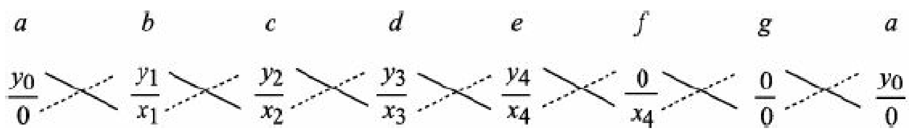
From the given distances and offsets, a point is selected as the origin. The coordinated of all other points are arranged with reference to the origin.



Taking g as the origin, the coordinates cf all other points are arranged as follows:

Points	Coordinates	
	X	Y
a	0	y_0
b	x_1	y_1
c	x_2	y_2
d	x_3	y_3
e	x_4	y_4
f	x_4	0
g	0	0
a	0	y_0

The coordinates are arranged in determinant form as follows:



Sum of products along the solid line,

$$\Sigma P = (y_0x_1 + y_1x_2 + \ldots + 0.0)$$

Sum of products, along the dotted line

$$\Sigma Q = (0.y_1 + x_1y_2 + \ldots + 0.y_0)$$

Required area = (1/2) (Σ P – Σ Q)

Example (AMIE S14, 8 marks)

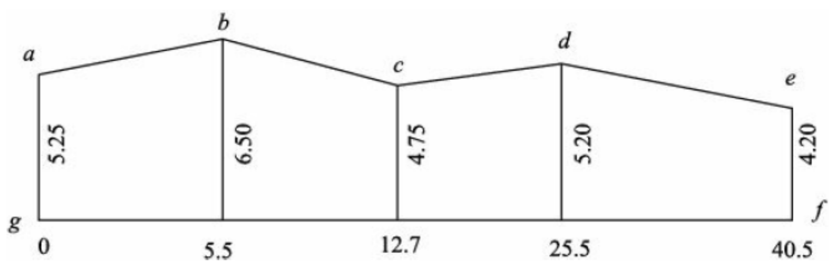
Following perpendiculars offsets were taken from chain line to a hedge:

Chainage, m	*0.0*	*5.5*	*12.7*	*25.5*	*40.5*
Offset, m	*5.25*	*6.50*	*4.75*	*5.20*	*4.20*

Calculate the area between the chain line and hedge by the co-ordinate method.

Solution

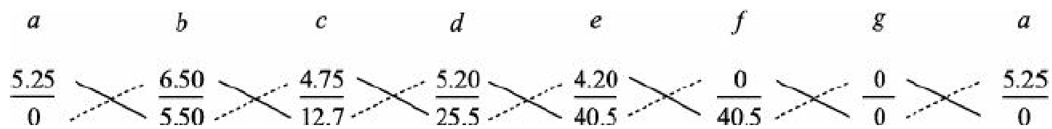
Figure



Making co-ordinate table

Points	Coordinates	
	X	Y
a	0	5.25
b	5.5	6.50
c	12.7	4.75
d	25.5	5.20
e	40.5	4.20
f	40.5	0
g	0	0
a	0	5.25

Determinant form



Sum of products along the solid line

$$\Sigma P = (5.25 \times 5.50 + 6.50 \times 12.7 + 4.75 \times 25.5 + 5.20 \times 40.5$$

$$+ 4.20 \times 40.5 + 0 \times 0 + 0 \times 0)$$

$$= 28.88 + 82.55 + 121.13 + 210.60 + 170.10 = 613.26$$

Sum of products along dotted line

$$\Sigma Q = (0 \times 6.50 + 5.50 \times 4.75 + 12.7 \times 5.20 + 25.5 \times 4.20$$

$$+ 40.5 \times 0 + 40.5 \times 0 + 0 \times 5.25)$$

$$= 26.13 + 66.04 + 107.10 = 199.27$$

Area

Required area = $(1/2)(\Sigma P - \Sigma Q) = (1/2)(613.26 - 199.27) = 206.995 \text{ m}^2$

Problem

The following perpendicular offsets were taken from a chain line to a hedge:

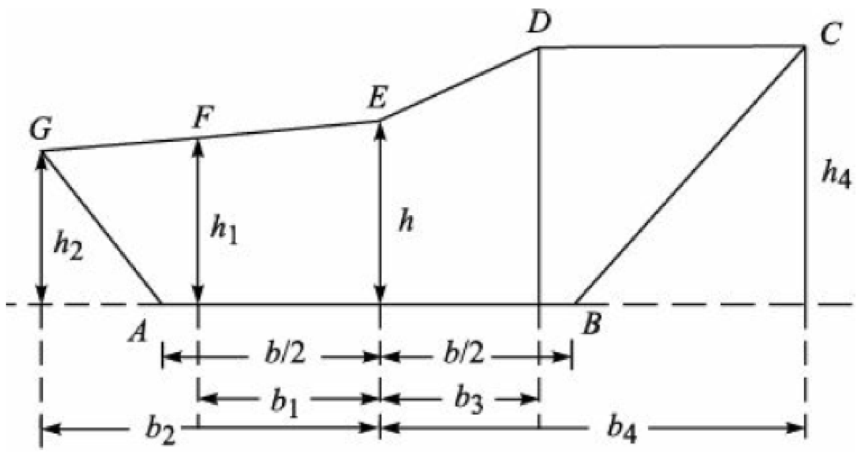
Distance (m)	*0*	*6*	*12*	*18*	*24*	*30*	*36*
Offset (m)	*5.40*	*4.50*	*3.60*	*2.70*	*1.80*	*2.25*	*3.15*

Calculate the area enclosed between the chain line and the offsets by (i) trapezoidal rule, and (ii) Simpson's rule.

Answer: 114.75 m^2, 114.3 m^2

COMPUTATION OF VOLUME

Figure



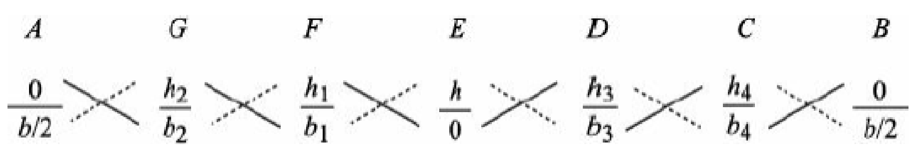
Area of multi level section

The cross-sectional data pertaining to an irregular section are noted in the following form:

Left	Centre	Right
$\frac{\pm h_2}{b_2}\ \frac{\pm h_1}{b_1}$	$\frac{\pm h}{0}$	$\frac{\pm h_3}{b_3}\ \frac{\pm h_4}{b_4}$

A positive sign in the numerator denotes a cut, and a negative sign indicates a fill.

Starting from the centre (E) and running outwards to the right and left, the coordinates of the vertices are arranged, irrespective of algebraic sign, in determinant form:



The sum of the products of the coordinates joined by solid lines is given by

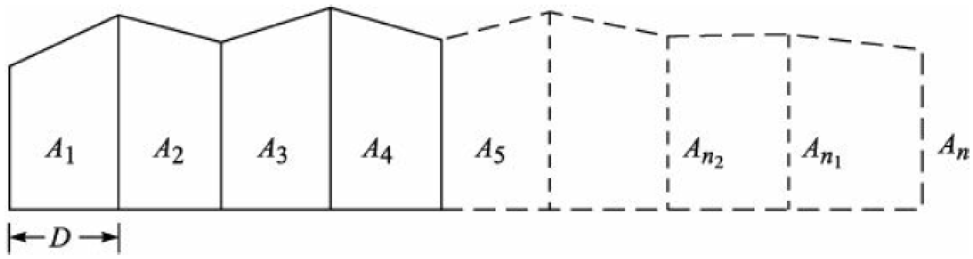
$$\Sigma P = h_3 \times 0 + h_4 \times b_3 + 0 \times b_4 + h_1 \times 0 + h_2 \times b_1 + 0 \times b_2$$

The sum of the products of the coordinates joined by dotted lines is given by

$$\Sigma Q = h \times b_3 + h_3 \times b_4 + h_4 \times (b/2) + h \times b_1 + h_1 \times b_1 + h_1 \times (b/2)$$

Area = $(\Sigma P \sim \Sigma Q)$

Calculation of Volume by trapezoidal rule



Volume = (Common distance/2) x {area of first section + area of last section

+ 2 (sum of area of other sections)}

Calculation of Volume by Prismoidal rule

V = (common distance/2) {area of first section + area of last section + 4(sum of areas of even sections) + 2 (sum of areas of odd sections)}

☞ *The prismoidal formula is applicable when there are an odd number of sections. If the number of sections is even, the end strip is treated separately and the area is calculated according to the trapezoidal rule. The volume of the remaining strips is calculated in the usual manner by the prismoidal formula. Then both the results are added to obtain the total volume.*

Example (Winter 2016, 5 marks)

The areas enclosed by the contours in a lake are as follows:

Contour (m)	*270*	*275*	*280*	*285*	*290*
Area (m^2)	*2,050*	*8,400*	*16,300*	*24,600*	*31,500*

Calculate the volume of water between the contours 270 m and 290 m by (a) the trapezoidal formula, and (b) the prismoidal formula.

Solution

Volume according to trapezoidal formula

V = (5/2) x {2,050 + 31,500 + 2(8,400 + 16,300 + 24,600)} = 330,375 m^3

Volume by prismoidal formula

V = {2,050 + 31,500 + 4(8,400 + 24,600) + 2(16,300)} = 330,250 m^3

Example

An embankment of 10 m width and side slopes 1½ : 1 is required to be made on a ground which is level in a direction transverse to the centre line. The central heights at 40 m intervals are as follows:

0.90, 1.25, 2.15, 2.50, 1.85, 1.35, and 0.85

Calculate the volume of earth work according to the trapezoidal formula.

Solution

Area, $\Delta = (b + sh) \times h$

$\Delta_1 = (10 + 1.5 \times 0.90) \times 0.90 = 10.22\ m^2$

$\Delta_2 = (10 + 1.5 \times 1.25) \times 1.25 = 14.84\ m^2$

$\Delta_3 = (10 + 1.5 \times 2.15) \times 2.15 = 28.43\ m^2$

$\Delta_4 = (10 + 1.5 \times 2.50) \times 2.50 = 34.38\ m^2$

$\Delta_5 = (10 + 1.5 \times 1.85) \times 1.85 = 23.63\ m^2$

$\Delta_6 = (10 + 1.5 \times 1.35) \times 1.35 = 16.23\ m^2$

$\Delta_7 = (10 + 1.5 \times 0.85) \times 0.85 = 9.58\ m^2$

Volume according to trapezoidal formula:

$$V = (40/2) \times \{10.22 + 9.58 + 2(14.84 + 28.43 + 34.38 + 23.63 + 16.23)\}$$

$$= 20\{19.80 + 235.02\} = 5{,}096.4\ m^3$$

Example

A railway embankment of formation width of 8 m and 2:1 side slope is to be constructed. The ground level along the centre line is as follows:

Chainage	*0*	*50*	*100*	*150*	*200*	*250*
GL (m)	*115.25*	*114.75*	*116.80*	*115.20*	*118.50*	*118.25*

The embankment has a rising gradient of 1 in 100, and the formation level at zero chainage is 115.00. Assuming the ground is level across the centre line, compute the volume of earth work.

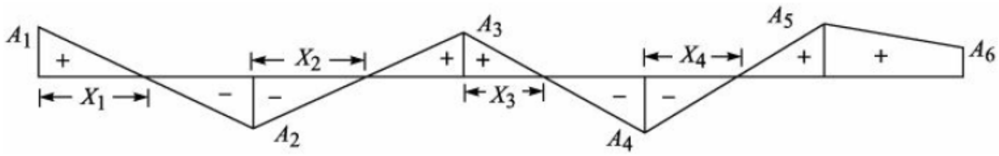
Solution

Making a cutting and filling table

Rise per 50 m = 50/100 = 0.50 m

Chainage	GL	FL	Cutting (+)	Filling (-)	Section
0	115.75	115.00	0.75		A_1
50	114.35	115.00		1.15	A_2
100	116.80	116.00	0.80		A_3
150	115.20	116.50		1.30	A_4
200	118.50	117.00	1.50		A_5
250	118.25	117.50	0.75		A_6

Figure



Calculation of x_1, x_2 etc.

$$\frac{x_1}{0.75} = \frac{50 - x_1}{1.15} \Rightarrow x_1 = 19.74\,m$$

$$\frac{x_2}{1.15} = \frac{50 - x_2}{0.80} \Rightarrow x_2 = 29.44\,m$$

$$\frac{x_3}{0.80} = \frac{50 - x_3}{1.30} \Rightarrow x_3 = 19.05\,m$$

$$\frac{x_4}{1.30} = \frac{50 - x_4}{1.50} \Rightarrow x_4 = 23.21\,m$$

Calculation of area

A = (b + sh) h

$A_1 = (8 + 2 \times 0.75) \times 0.75 = 7.13\ m^2$

$A_2 = (8 + 2 \times 1.15) \times 1.15 = 11.85\ m^2$

$A_3 = (8 + 2 \times 0.80) \times 0.80 = 7.68\ m^2$

$A_4 = (8 + 2 \times 1.30) \times 1.30 = 13.78\ m^2$

$A_5 = (8 + 2 \times 1.50) \times 1.50 = 16.50\ m^2$

$A_6 = (8 + 2 \times 0.75) \times 0.75 = 7.13\ m^2$

Calculation for volume

From chainage 0 to 50

Cutting = $\frac{7.13 + 0}{2} x 19.74 = 70.37\, m^3$

Filling = $\frac{0 + 11.85}{2} x 30.26 = 179.29\, m^3$

From chainage 50 to 100

Cutting = $\frac{11.85 + 0}{2} x 29.49 = 174.73\, m^3$

Filling = $\frac{0+7.68}{2} x 20.51 = 78.76 m^3$

From chainage 100 to 150

Cutting = $\frac{7.68+0}{2} x 19.05 = 73.15 m^3$

Filling = $\frac{0+13.78}{2} x 30.95 = 213.25 m^3$

From chainage 150 to 200

Cutting = $\frac{13.78+0}{2} x 23.2 = 159.92\ m^3$

Filling = $\frac{0+16.50}{2} x 26.8 = 221.02\ m^3$

From chainage 200 to 250

Cutting = $\frac{16.50+7.13}{2} x 50 = 590.75\ m^3$

Total cutting = 70.37 + 78.76 + 73.15 + 221.02 + 590.75 = 1034.05 m^3

Total filling = 179.29 + 174.73 + 213.55 + 159.92 = 727.19 m^3

Problem

The formation level of a road is at a constant RL of 150.00 m. The ground levels along the centre line of the road are as follows:

Chainage (m)	*0*	*40*	*80*	*120*	*160*	*200*	*240*
Ground level (m)	*152.60*	*151.90*	*149.00*	*150.90*	*151.50*	*152.45*	*151.20*

Compute the volume of earth work given that the formation width is 8 m and the side slope 2:1.

Answer: 3638.63 m^3

Problem

A railway embankment 600 m long has a formation level width of 11.5 m with side slope 2:1. If the ground and formation levels are as follows, calculate the volume of earth work. The ground is level across the centre line.

Distance (m)	*0*	*100*	*200*	*300*	*400*	*500*	*600*
GL (m)	*105.2*	*106.8*	*107.0*	*103.4*	*105.6*	*104.7*	*105.1*
FL (m)	*107.5*	*108.6*	*108.5*	*104.5*	*106.9*	*105.6*	*106.3*

Answer: 11,691.67 m^3—prismoidal, 12,091.51 m^3—trapezoidal

DISTORTED OR SHRUNK SCALES

Due to change in climatic conditions, the plans and maps generally get distorted. If no graphical scale is drawn on the plan, correct scale of the distorted plan (or map), may be calculated by the following method:

Step 1

Measure a distance between any two well defined points on the plan and calculate its corresponding ground distance from the scale i.e.,

1 cm = x metres. Let it be l metres.

Step 2

Measure the horizontal distance between the same points on the ground by chaining. Calculate the distance on plan with the scale. Let it be y cm.

Step 3

Calculate the shrinkage ratio or shrinkage factor which is equal to shrunk length / the actual length.

Step 4

Shrunk scale of plan = Shrinkage factor × Original scale.

Example

The area of a plot on a map is found, by planimeter, to be 10.22 cm^2. The scale of the map was 1:25000, but at present it is shrunk such that a line originally 5 cm on the map is now 4.8 cm. What is the correct field area in hectares ?

Solution

The area of the plot measured by the planimeter on the shrunk map = 10.22 cm^2 (Given)

The ratio of shrinkage = 4.8/5.0 = 0.96

The ratio of shrinkage of the area = $(0.96)^2$ = 0.9216

The area of the plot on unshrunk map on 1:25,000 scale = 10.22/0.9216 = 11.08941 cm^2

Area of 1 cm^2 on scale 1:25000 = 250 × 250 = 62500 m^2

$\therefore$ Area of 11.08941 cm on scale 1 : 25000 = 62500 × 11.08941

= 693088.12 m^2

= 69.3088 hectares.

Example

An old map was plotted to a scale of 40 m to 1 cm. Over the years, this map has been shrinking, and a line originally 20 cm long is only 19.5 cm long at present. Again, the 20 m chain was 5 cm too long. If the present area of the map measured by a planimeter is 125.50 cm^2, find the true area of the land surveyed.

Solution

According to the given conditions,

19.5 cm on the map was originally 20 cm.

Therefore, 1 cm on the map was originally (20/19.5) cm, and

1 cm^2 on the map was originally $(20^2/19.5^2)$ cm^2

125.50 cm^2 was originally $(20^2/19.5^2) \times 125.50 = 132.0184$ cm^2

Scale of map was 1 cm = 40 m

1 cm^2 = 1,600 m^2

Area on the ground = 1,600 × 132.0184 = 211,229.44 m^2

Since the chain was 0.05 m too long,

True area = $(20.05^2/20^2) \times 211{,}229.44 = 212{,}286.90$ m^2 = 21.2286 hectares

(1 hectare = 10,000 m^2)

Problem

The area of the plan of an old survey plotted to a scale of 10 metres to 1 cm measures now as 90.5 sq. cm as found by a planimeter. The plan is found to have shrunk so that a line originally 10 cm long, now measures 9.5 cm only. There was also a note on the plan that the 20 m chain used was 9 cm too short. Find the true area of the survey.

Answer: 9937.65 sq. m = 0.9938 hectares

ASSIGNMENT

Q.1. (AMIE W11, 16, S18, 10 marks): What is the principle of chain surveying? When this method is recommended. What are the various corrections to the tape measurement? Explain them.

Q.2. (AMIE W11, 6 marks): What are the principles of chain surveying? Describe how will you overcome the obstacles in chain surveying when chaining is obstructed but vision is free.

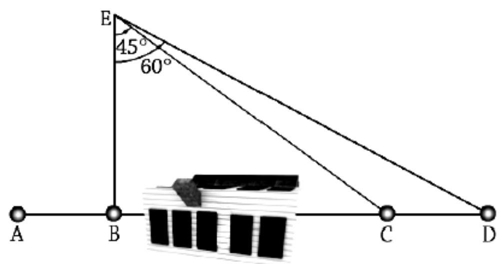
Q.3. (AMIE W16, 3 marks): Define "well conditioned triangle" in the process of chain surveying. In this connection define survey station, main station, subsidiary station and tie station. Give sketches.

Q.4. (AMIE W16, 3 marks): Explain the term "reconnaissance" and "index map" with the help of sketch.

Q.5. (AMIE W13): What is a scale factor? Where is it used ? What is the advantage of this scale factor?

Q.6. (AMIE W11, 6 marks): To continue a survey line AB past an obstacles, a line BC of 200 m long was set out perpendicular to AB, and from C angles BCD and BCE were set out of 60^0 and 45^0, respectively. Determine the lengths which must be chained off along CD and CE in order that ED may be in AB produced. Also, determine the obstructed length BE.

Solution: See following figure



L ABC is 90°

From A BCD, CD = BC sec 60^0 = 200 x 2 = 400 m

From A BCE, CE =BC sec45° = 200 x 1.4142 = 282.84 m

BC = BC tan 45° = 200 x 1 = 200 m

Q.7. (AMIE W15, 6 marks): The following perpendicular offsets were taken at 10 m intervals from a survey line to an irregular boundary line:

3.25 m; 5.60 m; 4.20 m; 6.65 m; 8.75 m; 6.20 m; 3.25 m; 4.20 m; 5.65 m

Calculate the area enclosed between the survey line, irregular boundary line, and first and last offsets by the application of (i) average ordinate rule (ii) trapezoidal rule (iii) Simpson's rule.

Answer: Trapezoidal rule: 433 sq. m; Simpson's rule: 439.67 sq-m

Q.8. (AMIE W16, 5 marks): Under noted readings are from a record of chain survey. Calculate the area enclosed by it.

To	B
17.10	144 m
18.90	126 m
15.30	108 m
9.90	90 m
7.20	72 m
7.20	54 m

11.70	36 m
15.30	18 m
24.30	0 m
From	A

Q.9. (AMIE S18, 6 marks): A river channel is 18 metres wide and the depth (Y) is of the water at distances (X) metres, from one bank are given in the following table. Find the area of the cross-section and average depth of water.

X	0	3	6	9	12	15	18
Y	1.5	2.1	4.5	6.3	9	4.8	18

Q.10. (AMIE W15, 7 marks): A railway embankment, 400 m long is 12 m wide at the formation level and has the side slopes 2 to 1. The ground levels at every 100 m along the centre line are as under:

Distance	0	100	200	300	400
RL	204.8	206.2	207.5	207.2	208.3

The formation level at 0 chainage (zero chainage) is 207.00 and the embankment has a rising gradient of 1 in 100. The ground is level across the centre line. Calculate the volume of earthwork.

Answer: Trapezoidal - 14137 m^3; Prismoidal - 14581 m^3

Q.11. (AMIE S13, 6 marks): The embankment of width 10 m and side slopes $1\frac{1}{2}:1$ is required to be made on a ground which is level in a direction transverse to the centre line. The central height at 40 m intervals are: 0.90, 1.25, 2.15, 2.50, 1.85, 1.35 and 0.85. Calculate the volume of earthwork according to the (i) trapezoidal rule (ii) prismoidal formula.

Answer: Trapezoidal - 5096.4 m^2; prismoidal - 5142.9 m^2

Q.12. (AMIE W16, 5 marks): A line 270 m long was measured with a 30 m long tape, which was 0.002 m too long and the error in making a tape length was ± 0.002 m. Calculate the amount of compensating and cumulative errors.

Q.13. (AMIE S17, 4 marks): The distance between two points measured with a 30 m chain was recorded as 216 m. It was afterwards found that chain was 10 cm too long. What is the true distance between the points.

Hint: L = 30 m; error in chain = 10 cm = 0.10 m, too long; incorrect length of chain = 30 + 0.10 = 30.10 m; measured length = 216 m; True length = (incorrect length/correct length) x 216 m

Q.14. (AMIE S17, 18, 4 marks): A survey was plotted to the scale of 1:2500. A certain area on a photo print reproduction was measured by a planimeter and was found to contain 100 square centimetres. The print shows a shrinkage of 1 % both up and down and along the sheet. Obtain the true area measured, expressing the result in square metres.

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